Non-Grassmann quantization of the massive Thirring model

Piotr Garbaczewski
Institute of Theoretical Physics, University of Wrocław, 50-205 Wrocław, Poland

(Received 22 April 1982; accepted for publication 29 July 1982)

A direct quantization of the c-number (semi) classical massive Thirring model in the inverse scattering formalism leads to the Bose massive Thirring model, which is equivalent to the conventional Fermi one, both having identical S-matrices and bound-state spectra.

PACS numbers: 03.70. + k, 03.65.Sq

A. There is a wide-spread belief among quantum field theorists that (semi) classical c-number spinor fields are unrelated to their quantized Fermi partners. A spectacular manifestation of this situation is the use of Grassmann algebra-valued spinor models as the would-be-the-only-reasonable pseudoclassical levels for Fermi systems. For example, the path integration methods if applied to spinor systems, by the very assumption do exclude the conventional path notion in the c-number function ring. An anticommuting, i.e., Grassmann algebra valued ring is then conventionally in use. From the practical point of view (perturbative calculations) this idea is quite justified, and it was consequently the main motivation for the studies of the Grassmann algebra valued massive Thirring model, which has been proved to be a completely integrable system.[1] There appeared, however, a problem of the quantization of this system via the quantum spectral transform method (which is successful for many other $1+1$ dimensional models). This quantization route which we call a *Grassmann quantization* of the massive Thirring model still remains uncompleted.

Quite the contrary, in the series of papers, Refs. 2–4, we have investigated the relationships between the (semi) classical c-number spinor systems and the respective quantum Fermi models, following the idea of Ref. 5 that the c-number solutions of the classical spinor field equations should have some relevance for the construction of the appropriate quantum field theory. In Refs. 2 and 3 we have demonstrated that the relationship exists provided the Fermi models admit a "bosonization" in terms of free Bose fields. In the practical application of Ref. 4 it means that the Fermi massive Thirring model admits three different types of the asymptotic (Haag) expansions, depending on the choice of the state space, and provided one takes into account spaces generated by soliton coherent states, see, e.g., Ref. 4.

The underlying expansions appear either in terms of the massive vector boson without the Proca constraint, or in terms of the neutral massive scalar (then the relationship with the sine-Gordon model can be established), and under special circumstances only, in terms of the free (asymptotic) two-component fermion. The latter case fits into the conventional asymptotic completeness condition, otherwise the fermion being confined.

Because in the light of Ref. 4 there exists an indirect relationship of the c-number massive Thirring model (MT) to the Fermi MT, it is quite natural to state a problem of the direct quantization of the c-number massive Thirring model. This route we call a *non-Grassmann quantization* of the MT. We accomplish this quantization in the quantum inverse transform formalism of Ref. 6, by exploiting both the results of Refs. 7 and 8 concerning the complete integrability of the c-number MT and those on the quantization of the sine-Gordon model.[9]

We demonstrate that in the quantum inverse method, the Bose quantized MT has a lattice approximation, which is equivalent to that of the quantum sine-Gordon model. By repeating the arguments of Ref. 9, one is then capable of deriving a continuum limit in which both models have the bound-state spectrum (and the S-matrix) identical to this of the conventional Fermi MT.

B. The classical (semiclassical in fact) c-number massive Thirring model is known to be a completely integrable system.[7,8] The field equation

$$\begin{gathered}(-i\gamma_\nu\partial^\nu+m)\psi=g\gamma^\nu\psi(\bar\psi\gamma_\nu\psi),\quad \nu=0,1,\ g>0\,,\\ \gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \gamma^1=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad \gamma^5=\gamma^0\gamma^1\,,\\ \bar\psi=\psi^*\gamma^0\,.\end{gathered}\tag{1}$$

can be rewritten as the system

$$\begin{gathered}-i\psi_{1t}-i\psi_{1x}+m\psi_2+2g|\psi_2|^2\psi_1=0\,,\\ -i\psi_{2t}+i\psi_{2x}+m\psi_1+2g|\psi_1|^2\psi_2=0\,,\end{gathered}\tag{2}$$

which is known to admit classical (c-number spinors) soliton solutions.[8] An equivalent description of Eq. (2) is known to be provided by the commutator $[X,T]_-=0$ of the two objects:

$$\begin{aligned}X&=2i\partial_x+g(\bar\psi\gamma_1\psi)\gamma^5+(2mg)^{1/2}\\&\quad\times\begin{pmatrix}0,\ \lambda\psi_2^*-\lambda^{-1}\psi_1^*\\ \lambda\psi_2-\lambda^{-1}\psi_1,\ 0\end{pmatrix}-\frac{m}{2}(\lambda^2-\lambda^{-2})\gamma^5\,,\\ T&=2i\partial_t+g(\bar\psi\gamma_0\psi)\gamma^5+(2mg)^{1/2}\\&\quad\times\begin{pmatrix}0,\ \lambda\psi_2^*+\lambda^{-1}\psi_1^*\\ \lambda\psi_2+\lambda^{-1}\psi_1,\ 0\end{pmatrix}-\frac{m}{2}(\lambda^2+\lambda^{-2})\gamma^5\,,\end{aligned}\tag{3}$$

i.e., by the condition that all terms standing in the commutator at different powers of the spectral parameter λ do vanish. For $X=X(\lambda)$, we shall adopt the form

 0022-2488/83/071806-04$02.50

$$X' = \tfrac{1}{2} iX = -\partial_x + L(x,\lambda),$$

$$L(x,\lambda) = i\begin{pmatrix} \tfrac{1}{4}m(\lambda^{-2}-\lambda^{2}) + g(\rho_1-\rho_2), & (\lambda\psi_2^* - \lambda^{-1}\psi_1^*)(mg/2)^{1/2} \\ (mg/2)^{1/2}(\lambda\psi_2 - \lambda^{-1}\psi_1), & \tfrac{1}{4} m(\lambda^{2}-\lambda^{-2}) - g(\rho_1-\rho_2) \end{pmatrix}, \tag{4}$$

$$\rho_i = |\psi_i|^2, \quad i = 1,2$$

According to Ref. 6, a straightforward quantized version of the problem(1) appears if one replaces the classical fields $\psi_i(x)$ by the quantum operators $\hat{\psi}_i(x)$, satisfying the (equal $t=0$ time) canonical commutation relations, not the canonical anticommutation ones (CAR) as demanded by convention:

$$[\hat{\psi}_i(x), \hat{\psi}_j^*(y)]_- = \alpha\delta_{ij}\delta(x-y), [\hat{\psi}_i(x), \hat{\psi}_j(y)]_- = 0, \tag{5}$$

provided we make a change in $L(x,\lambda)$; $L(x,\lambda) \to \hat{L}(x,\lambda)$,

$$\hat{L}(x,\lambda) = i\begin{pmatrix} \tfrac{1}{4}m(-\lambda^{2}-\lambda^{-2}) + g(-\xi\hat{\rho}_1 + \eta\hat{\rho}_2), & (mg/2)^{1/2}(\lambda\hat{\psi}_2^* - \lambda^{-1}\hat{\psi}_1^*) \\ (mg/2)^{1/2}(\lambda\hat{\psi}_2 - \lambda^{-1}\hat{\psi}_1), & \tfrac{1}{4}m(\lambda^{2}-\lambda^{-2}) + g(\eta\hat{\rho}_1 - \xi\hat{\rho}_2) \end{pmatrix}. \tag{6}$$

$$\hat{\rho}_i(x) = \hat{\psi}_i^*(x)\hat{\psi}_i(x), \quad i=1,2, \quad \eta = \exp\gamma/\cosh\gamma,$$

$$\xi = \exp(-\gamma)/\cosh\gamma, \quad \gamma = \tfrac{1}{2}i \arcsin\alpha, \alpha\in R.$$

With the operator valued matrix $\hat{L}(x,\lambda)$ in hand, let us introduce the tensor product matrices

$$L' = \hat{L}\otimes I, \quad L'' = I\otimes\hat{L} \tag{7}$$

according to the rule

$$F\otimes G = \begin{pmatrix} f_{11}G, & f_{12}G \\ f_{21}G, & f_{22}G \end{pmatrix}. \tag{8}$$

Then the matrix equation

$$R(\lambda,\lambda')L'(x,\lambda)L''(x,\lambda') = L''(x,\lambda')L'(x,\lambda)R(\lambda,\lambda'), \tag{9}$$

can be solved by means of the 4×4 matrix $R = R(\lambda,\lambda')$, with the c-number matrix elements,[4]

$$R = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & a \end{pmatrix}, \quad \begin{aligned} a &= 1, \\ b &= \sinh 2\gamma/\sinh(u+2\gamma), \\ c &= \sinh u/\sinh(u+2\gamma), \end{aligned} \tag{10}$$

where $\exp\ u = \lambda/\lambda' = \exp(v-v')$, $\exp v = \lambda$, $\exp v' = \lambda'$. Let us notice that the change of variables in (6),

$$v\to v-\gamma, \ v'\to v'-\gamma \tag{11}$$

does not affect the R-matrix (10) because $u = v - v' \to u$. Notice that (11) corresponds to the replacement $\lambda \to \lambda\exp(-\gamma)$. We shall adopt a bit more sophisticated version of (11), namely,

$$v\to v-(\gamma - i\pi/4), \quad v'\to v' - (\gamma - i\pi/4). \tag{12}$$

Recall that the parameter γ is purely imaginary: $\gamma = i\mu/2$, $\mu = \arcsin\alpha$. Consequently we arrive at

$$\hat{L}(x,\lambda) \to i\begin{pmatrix} -\tfrac{1}{2}m\sinh[2v - i(\mu-\pi/2)] + g(e^{i\mu/2}\hat{\rho}_2 - e^{-i\mu/2}\hat{\rho}_1), & (mg/2)^{1/2}(e^{v}\hat{\psi}_2^* - e^{-v}\hat{\psi}_1^*) \\ (mg/2)^{1/2}(e^{v}\hat{\psi}_2^* - e^{-v}\hat{\psi}_1^*), & \tfrac{1}{2}m\sinh[2v - i(\mu-\pi/2)] + g(e^{i\mu/2}\hat{\rho}_1 - e^{-i\mu/2}\hat{\rho}_2) \end{pmatrix}, \tag{13}$$

$$c = c(\lambda,\lambda') = \sinh(v-v')/\sinh(v-v'+i\mu),$$

$$b = (\lambda,\lambda') = i\sin\mu/\sinh(v-v'+i\mu),$$

$$\lambda = \exp v, \quad \lambda' = \exp v'.$$

C. In general how to apply the quantum spectral transform method on the continuum level is not straightforward. Usually one adopts some discretization scheme, like that in Ref. 9, where the quantum inverse scattering formalism as a basic ingredient includes a matrix equation:

$$X\Psi = \left(\frac{\partial}{\partial x} + iQ\right)\Psi = 0, \tag{14}$$

with $Q = Q(x) = i\hat{L}(x,\lambda)$. Its discretized version on a linear lattice of length L and spacing δ, $N = L/\delta$ reads

$$\Psi_{n+1} = L_n(x)\Psi_n,$$

$$L_n(\lambda) = I + i\int_{x_n}^{x_n+\delta} Q(z)\,dz = I - \int_{x_n}^{x_n+\delta} L(z,\lambda)\,dz, \tag{15}$$

$$x_m = -L/2 + n\delta, \quad n = 0,1,\ldots,N, \quad N = L/\delta.$$

In particular one finds

$$\Psi_{L/\delta} = \Psi_N := T(\lambda) = \overleftarrow{\prod_{n=0}^{N-1}} L_n(\lambda) = L_{N-1}(\lambda)\ldots L_0(\lambda). \tag{16}$$

The so defined transition operator for an interval $L = N\delta$,

$$T(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}, \tag{17}$$

$$R(\lambda,\lambda')(T(\lambda) \otimes T(\lambda')) = (T(\lambda') \otimes T(\lambda))R(\lambda,\lambda'),$$

is a fundamental object of the quantum inverse method. Upon discretization of (15), one represents $A(\lambda), B(\lambda), C(\lambda), D(\lambda)$ by operators in the $2N$ particle Hilbert space $\mathscr{H}_{2N} = \Pi_{i=1}^{N} \otimes (h \otimes \tilde{h})_i$ carrying a ($2N$ particle) Fock representation of the CCR algebra:

$$\psi_i(n)\Omega = 0, \ \forall i = 1,2, \ n = 1,2,...,N, \quad \Omega = \prod_{i=1}^{N} \otimes (\omega_0 \otimes \tilde{\omega}_0)_i \ ,$$

$$[\psi_i(n), \psi_j^*(m)]_- = \delta_{ij}\delta_{mn}\cdot\alpha, \quad [\psi_i(n), \psi_j(m)]_- = 0, \tag{18}$$

$$\psi_i(n) = \frac{1}{\sqrt{\delta}} \int_{R^1} \psi_i(x)\chi_n(x)\, dx,$$

$$\chi_n(x) = \begin{cases} 1, & x \in [x_n, x_n + \delta], \\ 0, & x \in [x_n, x_n + \delta]. \end{cases}$$

In particular we can consider the action of matrix elements of the operator $L_n(\lambda)$ on the Fock vacuum Ω. One immediately verifies that

$$\begin{aligned} L_{n21}\Omega &= 0, \\ L_{n11}\Omega &= \{1 - \tfrac{1}{2}im\delta \sinh[2v - i(\mu - \pi/2)]\}\Omega \\ &= \{1 + \tfrac{1}{2}m\delta\cosh(2v - i\mu)\}\Omega \\ &\cong \exp[\tfrac{1}{2}m\delta\cosh(2v - i\mu)]\cdot\Omega \\ &= \exp[a(\lambda)\cdot\delta]\cdot\Omega. \end{aligned} \tag{19}$$

In the above we use an identity
$\sinh[(2v - i\mu) + i\pi/2] = i\cosh(2v - i\mu)$,
Analogously,

$$L_{n22}\Omega \cong \exp[d(\lambda)\cdot\delta]\cdot\Omega = \exp\{-\tfrac{1}{2}m\delta\cosh(2v + i\mu)\}\Omega, \tag{20}$$

and consequently,

$$\exp[a(\lambda) + d(\lambda)]\delta = \exp(im\delta\cdot\sin\mu\cdot\sinh 2v), \tag{21}$$

i.e.,

$$\exp[a(\lambda) + d(\lambda)]N = \exp ikL = (\exp ik\delta)^{L/\delta}, \tag{22}$$

$$k = (m\sin\mu)\cdot\sinh 2v,$$

with [make a product of matrices L_n according to (16)]:

$$A(\lambda)\Omega = \exp[a(\lambda)N]\cdot\Omega, \quad D(\lambda)\Omega = \exp[d(\lambda)N]\cdot\Omega. \tag{23}$$

Hence in addition to the R-matrix (13), we have specified the reference (Fock) state Ω solving the eigenvalue problem for $A(\lambda), D(\lambda)$, Eqs. (19)–(23) and being annihilated by $C(\lambda)$. These data completely suffice to specify a representation of the algebra of A, B, C, D operators as defined by the commutation relation (17). Then we can construct the eigenvectors of the transfer operator

$$\mathrm{Tr}T(\lambda) = \mathscr{T}(\lambda) = A(\lambda) + D(\lambda), \tag{24}$$

as follows

$$|\lambda_1,...\lambda_n) = \prod_{i=1}^{n} B(\lambda_i)\Omega, \tag{25}$$

provided we have satisfied the periodicity condition

$$\exp ik_i L = \prod_{j=n}^{n} \frac{\sinh(v_i - v_j + i\mu)}{\sinh(v_i - v_j - i\mu)} \tag{26}$$

$$k_i = (m\sin\mu)\cdot\sinh 2v_i, \quad v_i = \ln\lambda_i, \quad i = 1,...,n.$$

The respective eigenvalue reads

$$\mathscr{T}(\lambda)|\lambda_1,...,\lambda_n) = \Lambda(\lambda,\lambda_1,\lambda_2,...,\lambda_n)|(\lambda_1,...,\lambda_n), \tag{27}$$

$$\Lambda(\lambda,\lambda_1,...,\lambda_n) = \exp[a(\lambda)L] \prod_{j=1}^{n} \frac{1}{c(\lambda_j,\lambda)} + \exp[d(\lambda)L] \prod_{j=1}^{n} \frac{1}{c(\lambda,\lambda_j)}$$

with $c(\lambda,\lambda')$ given by (13).

D. By recalling Ref. 9 we find that upon a mere identification [compare, e.g., (1.29) in Ref. 9],

$$m_{\mathrm{sG}}^2\cdot\delta/4 = m_{\mathrm{MT}}, \tag{28}$$

the above representation becomes isomorphic with this found for the quantum sine-Gordon model on a lattice. Obviously letting $\delta \to 0$ (continuum limit) must be accompanied by $m_{\mathrm{sG}} \to \infty$ to keep m_{MT} finite. The $m_{\mathrm{sG}} \to \infty$ demand is quite natural in the light of our previous analysis of the relationships between the sine-Gordon and xyz Heisenberg models.[10,11] These two models can be considered as equivalent in the continuum limit, upon the lattice identification analogous to that of (28):

$$l'^2/16\delta = m_{\mathrm{sG}}^2\cdot\delta/4 \tag{29}$$

of the xyz model parameter l' (an elliptic modulus of Jacobi theta functions), see, e.g. Refs. 11 and 12 and the sine-Gordon coupling constant m_{sG}, where $\delta \to 0$ means both $m_{\mathrm{sG}} \to \infty$ and $l' \to 0$ (the weak anisotropy limit of Ref. 13.).

In the above discussion one must, however, remember that the Bose MT algebra (17) is represented in the Hilbert space $\mathscr{H}_N = \Pi_{i=n}^{N} {}^{\otimes}(h \otimes \tilde{h})_i$, while this for the sine-Gordon system in $\mathscr{H}_N = \Pi_{i=1}^{N} {}^{\otimes} h_i$, and this for the xyz model can be represented in a proper subspace $P\mathscr{H}_N = \Pi_{i=1}^{N} {}^{\otimes}(ph)_i$ of $\mathscr{H}_N$, with p being a two-level projection of Ref. 10 in h. $P\mathscr{H}_N$ can be equivalently rewritten as $\Pi_{i=n}^{N} {}^{\otimes}(C_2)_i$, where C_2 is a two-dimensional vector space.

On the lattice level both the Bose MT and sine-Gordon representations of the algebra (17) are equivalent and both become equivalent to the representation of the xyz Heisenberg model algebra in the continuum limit. In this case the Coleman's equivalence with the Fermi MT is a straightforward consequence.

E. With respect to the mass spectrum or the S-matrix arising in the continuum limit of the above models, the (Coleman's) equivalence of the Bose MT and the Fermi MT is guaranteed by the lattice identification of the Bose MT with the sine-Gordon model in the quantum inverse method. The procedure of Ref. 9 allows then the recovery of a continuum limit for the spectrum of the lattice models.

A few words should be said about the related quantum fields. One knows that while passing from the *xyz* model to the Fermi MT, there is a natural way to recover Fermi fields from renormalized lattice spin 1/2 degrees.[10–14] However, if one starts from the lattice Bose models,[10] like the above sine-Gordon or Bose MT, the emergence of fermions is not apparent at all. The lesson of Refs. 4, 10, and 11 in this context is that these lattice models can be constrained via the so called spin 1/2 approximation to the *xyz* model. A continuum limit of such a projected lattice Bose model gives the *S*-matrix and the spectrum identical to that of the sine-Gordon/Fermi MT models. However, in contrast to the full Bose MT, the resulting state space is precisely the space of Fermi states of the quantum Bose field, see, e.g. Refs. 4 and 11.

On such a space the irreducible Fermi fields can be consistently defined. Certainly the Bose MT can be rewritten as the *reducible Fermi model*. For 1 + 1 dimensional models, a formal relationship with the spin (1/2) *xyz* Heisenberg model can be introduced by means of the previously defined projection *P*:

$$H_B = PH_BP + PH_B(1-P) + (1-P)H_BP + (1-P)H_B(1-P), \qquad (30)$$

where (this is a spin 1/2 approximation constraint)

$$PH_BP \equiv H_{xyz}. \qquad (31)$$

For the sine-Gordon system in the continuum limit one arrives,[10,11] at the property rather rarely realized for lattice Bose systems:

$$H_B \equiv H_{xyz} + (1-P)H_B(1-P), \quad [H_B, P]_- = 0, \qquad (32)$$

which is in fact another version of the equivalence statement for the *xyz* and sine-Gordon models on the appropriate (a continuum limit of $P\mathscr{H}_N$) state space. The procedure of Ref. 11 with slight modifications can be repeated for the Bose MT, to prove that the formula (32) is valid in the continuum limit of the Bose MT. However, now the starting lattice Hilbert space of interest is $\mathscr{H}_{2N}$ and

$$P\mathscr{H}_{2N} = P\prod_{i=1}^{N}{}^{\otimes}(h\otimes\tilde h)_i := P\prod_{i=1}^{N}{}^{\otimes}(h_{2i-1}\otimes h_{2i}) = \prod_{i=1}^{N}{}^{\otimes}[(ph)_{2i-1}\otimes(ph)_{2i}] = \prod_{i=1}^{2N}{}^{\otimes}(ph)_i, \qquad (33)$$

where p is a two level projection of Ref. 10 in the single particle Hilbert space h.

If we start from the lattice CCR algebra generators associated with (18)

$$[a_u, a_i^*] = \delta_{ui}, [\tilde a_u, \tilde a_i^*]_- = \delta_{ui}, a_u\Omega = 0 = \tilde a_u \Omega \ \forall k, \quad [a_u^\#, \tilde a_i^\#]_- = 0 = [a_u, a_i]_- = [a_u^*, a_i^*]_-, \qquad (34)$$

the underlying projections are

$$P_u = :\exp(-a_u a_u): + a_u^*:\exp(-a_u^* a_u):a_u \quad (\text{in } h), \quad \widetilde P_u = :\exp(-\tilde a_u\tilde a_u): + \tilde a_u^*:\exp(-\tilde a_u^*\tilde a_u):\tilde a_u \quad (\text{in } \tilde h), \qquad (35)$$

$$P = \prod_{u=1}^{N}(p_u\cdot\tilde p_u)$$

and one easily checks that

$$Pa_u^*P \equiv \sigma_u^+, \quad Pa_uP \equiv \sigma_u^-, \quad P\tilde a_u^*P \equiv \tilde\sigma_u^+, \quad P_{\tilde u}P \equiv \tilde\sigma_u^-, \qquad (36)$$

determine the spin 1/2 SU(2) group generators for the linear chain of spins 1/2. Upon the change of labelling $\{\sigma_u^\pm, \tilde\sigma_u^\pm\}_{u=1,\dots,N} \rightarrow \{\sigma_u^\pm\}_{u=1,\dots,2N}, \{\sigma_{2i-1}^\pm, \tilde\sigma_u^\pm = \sigma_{2i}^\pm\}_{i=1,2,\dots,N}$ being newly introduced, an application of the Jordan–Wigner transformation allows us to convert a $2N$ site spin 1/2 system into the $2N$ component Fermi system. This step was carefully investigated in Ref. 14.

ACKNOWLEDGMENT

I have received a copy of Ref. 6 prior to publication from Professor L. D. Faddeev to whom I am greatly indebted.

[1] A. G. Izergin and P. P. Kulish, Theor. Mat. Phys., 189 (1980).
[2] P. Garbaczewski, J. Math. Phys. **19**, 642 (1978).
[3] P. Garbaczewski, J. Math. Phys. **23**, 442 (1982).
[4] P. Garbaczewski, "Quantization of spinor fields. III. Fermions on coherent (Bose) domains," J. Math. Phys., (to be published) and Ann. Phys. N.Y. **139**, 293 (1982).
[5] R. Jackiw, Rev. Mod. Phys. **49**, 681 (1977).
[6] P. P. Kulish and E. K. Sklyanin, *Quantum Spectral Transform Method. Recent Developments*, Springer Lecture Notes in Physics, Vol. 151, edited by J. Hientarinta and J. Mickelsson (Springer-Verlag, Berlin, 1982).
[7] A. V. Mikhaĭlov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 356 (1976) [JETP Lett. **23**, 320 (1976)].
[8] E. A. Kuznetsov and A. V. Mikhaĭlov, Teor. Mat. Fiz. **30**, 323 (1977).
[9] E. K. Sklyanin, L. A. Tahtajan and L. D. Faddeev, Teor. Mat. Fiz. **40**, 194 (1979).
[10] P. Garbaczewski, J. Math. Phys. **22**, 574 (1981).
[11] P. Garbaczewski, "Mechanisms of the "fermion–boson reciprocity," J. Math. Phys, (to be published).
[12] H. Bergknoff and H. B. Thacker, Phys. Rev. D **19**, 3666 (1979).
[13] A. Luther, Phys. Rev. B **14**, 2153 (1976).
[14] M. Lüscher, Nucl. Phys. B **117**, 425 (1976).